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Weak decays to final states with Radial Excitation Admixtures

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Abstract

We consider the weak decays of a B meson to final states that are mixtures of S-wave radially excited components. We consider non leptonic decays of the type $B \to \rho'\pi/B \to \rho\pi$, $B \to \omega'\pi/B \to \omega\pi$ and $B \to \phi'\pi/B \to \phi\pi$ where ρ', ω' and ϕ' are higher ρ, ω and ϕ resonances. We find such decays to have larger or similar branching ratios compared to decays where the final state ρ, ω and ϕ are in the ground state. We also study the effect of radial mixing in the vector and the pseudoscalar systems generated from hyperfine interaction and the annihilation term. We find the effects

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of radial mixing to be small and generally negligible for all practical purposes in the vector system. However, in the $\eta-\eta'$ system the effects of radial mixing are appreciable and seriously affect decay branching ratios for $B \to \eta(\eta')K(K^*)$. In particular we find that nonstandard $\eta(\eta')$ mixing can resolve the puzzles in $B \to \eta(\eta')K$ decays.

1 Introduction

Nonleptonic B decays play a very important role in the study of CP violation. It is expected that these studies will test the standard model(SM) picture of CP violation or provide hints for new physics. Most studies of two body nonleptonic B decays have concentrated on processes of the type $B \to M_1 M_2$ where both M_1 and M_2 are mesons in the ground state configuration. Here we want to look at nonleptonic B decays to final states where one of the final state meson contain admixtures of radially excited components. We expect such decays to have larger or similar branching ratios compared to decays where the final state contains the same meson in the ground state. There is an easy explanation for such a statement. For simplicity let us consider a simple model in which $B \to \pi M$ and M is a simple flavor eigenstate with no flavor mixing beyond isospin; e.g. ρ, ω or ϕ and we are interested in comparing the branching ratios for the final states containing the ground state configuration of meson M and its radial excitation. Some possible examples are:

$$BR(\bar{B}^0 \to \pi^- \rho^{+\prime})/BR(\bar{B}^0 \to \pi^- \rho^+) \tag{1}$$

$$BR(B^- \to \pi^- \omega')/BR(B^- \to \pi^- \omega) \tag{2}$$

$$BR(B_s \to \pi^0 \phi')/BR(B_s \to \pi^0 \phi) \tag{3}$$

where $\rho^{+\prime}$, ω' and ϕ' are radially excited states.

We assume an extreme factorization approximation in which the b quark decays into a pion and a u-quark and we neglect the relative Fermi momentum of the initial b and the spectator $\bar{q}$. The quark transition for the processes (1) and (2) is then

$$b \to \pi^-(\vec{p}) u(-\vec{p}) \tag{4}$$

where $\vec{p}$ denotes the final momentum of the π^-. For the process (3) the quark transition is essentially similar to the one above

$$b \to \pi^0(\vec{p}) s(-\vec{p}) \tag{5}$$

where now $\vec{p}$ denotes the final momentum of the π^0.

Concentrating on the processes (1) and (2) the transition matrix for the full decay has the form

$$\left\langle \pi^-(\vec{p}) M(-\vec{p}) \right| T \left| B \right\rangle = \left\langle M(-\vec{p}) \right| F \left| u(-\vec{p}) \bar{q}(0) \right\rangle \cdot \left\langle \pi^-(\vec{p}) u(-\vec{p}) \right| W \left| b \right\rangle \tag{6}$$

where T denotes the transition matrix for the hadronic decay which factors into a weak matrix element at the quark level denoted by W and a fragmentation matrix element denoted by F describing the transition of a quark with momentum $-\vec{p}$ and an antiquark with zero momentum to make a meson with momentum $-\vec{p}$.

It is immediately clear that if the final momentum $\vec{p}$ is large, the fragmentation matrix element will depend upon the high momentum tail of the meson wave function. This might tend to favor radial excitations over ground states, since the radial excitations are expected to have higher kinetic energies. We now note that the harmonic oscillator wave functions commonly used in hadron spectroscopy have a Gaussian tail for their high momentum components and this can suppress the fragmentation matrix element $\langle M(-\vec{p})| F |u(-\vec{p})\bar{q}(0)\rangle$ in comparison with wave functions from a different confining potential. Hence the branching ratio to a final state which is radially excited will be sensitive to the choice of the confining potential.

So far we have assumed the physical states to be pure radial excitations. However additional interactions can mix the various radial excited components. For instance hyperfine interactions can mix radial excitations with the same flavor structure and so in general in the ρ, ω and ϕ system the various physical states will be admixtures of radial excitations [1, 2]. Flavor mixing in the vector system is known to be small but is important in the pseudoscalar sector. Here the mixing in the $\eta(\eta')$ system receives an additional significant contribution from the annihilation diagram that leads to flavor mixing of the strange and non strange parts of the $\eta(\eta')$ wavefunction. The mixing in the pseudoscalar sector, therefore, is different from the ideal mixing found in the $\omega - \phi$ system. It is also possible for the annihilation term to mix states that are radial excitations allowing the the $\eta(\eta')$ wavefunction to contain radially excited components . Such non standard $\eta(\eta')$ mixing can have important implication for the non leptonic decays $B \to \eta(\eta')K(K^*)$.

In the transitions chosen in (1),(2) and (3) the radially excited meson must include the spectator quark and therefore must depend upon the fragmentation matrix element $\langle M(-\vec{p})| F |u(-\vec{p})\bar{q}(0)\rangle$. For the case of $B \to \eta(\eta')K$ decays, factorization results in the kaon leaving the weak vertex with its full momentum and the remaining quark carries the full momentum of the final meson. There is therefore a form factor in which there is a large internal momentum transfer needed to hadronize this quark with the spectator antiquark. This might favor the radial excitation if it has a much higher mean internal momentum. However things are more complicated here as the $\eta(\eta')$ can also be produced by an $q\bar{q}$ pair in a penguin diagram without containing the spectator quark. One possibility is when the $\bar{s}$

quark in the QCD penguin combines with the s quark from the $b \to s$ transition to form the $\eta(\eta')$. Another possibility is when a $\bar{q}$ and q pair (where $q = u, d, s$), appearing in the same current in the effective Hamiltonian, hadronizes to the $\eta(\eta')$. In the diagrammatic language this is often represented as a "gluon" splitting into a $q\bar{q}$ pair which then hadronizes into a $\eta(\eta')$. This term is usually called OZI suppressed [3, 4] as in most decays the contribution of this term is indeed suppressed with respect to other terms in the decay amplitude. This may not be the case in the $B \to \eta(\eta')K$ decays where the OZI term can be of comparable size as other terms in the decay amplitude and in particular we show that $B \to \eta' K^*$ can have significant contribution from the OZI suppressed term.

This paper is organized in the following manner: In the next section we study the mass mixing in the vector meson sector, involving the ρ, ω and ϕ, and the pseudoscalar sector involving the $\eta(\eta')$. In section 3 we present a general treatment of nonleptonic B decays using the effective Hamiltonian and the factorization assumption. We then show how this approach is related to the diagrammatic approach of studying nonleptonic B decays. In section 4 we study nonleptonic decays of B to final states involving higher resonant ρ, ω and ϕ states. This is followed by section 5 where we make predictions for $B \to \eta(\eta')K$ and $B \to \eta(\eta')K^*$ and comment on the relevance of the OZI suppressed term in the calculation of these decays. Finally we present our conclusions.

2 Mass mixing in the vector meson and pseudoscalar sector

We start with the mixing for the ρ system. To obtain the eigenstates and eigenvalues we diagonalize the mass matrix which has the form

$$< q'_a \bar{q}'_b, n' | M | q_a \bar{q}_b, n > \;=\; \delta_{aa'}\delta_{bb'}\delta_{nn'}(m_a + m_b + E_n) + \delta_{aa'}\delta_{bb'}\frac{B}{m_a m_b}\vec{s}_a \cdot \vec{s}_b \psi_n(0)\psi_{n'}(0) \quad (7)$$

where $\vec{s}_{a,b}$ and $m_{a,b}$ are the quark spin operators and masses. Here $n = 0, 1, 2$ and the basis states are chosen as $|N >= |u\bar{u} + d\bar{d} > /\sqrt{2}$ and $S = |s\bar{s} >$. In the above equation E_n is the binding energy of the n^{th} radially excited state and B is the strength of the hyperfine interaction. Note we are only presenting results for the neutral mesons. A similar treatment also can be used for the charged mesons. We will use a very simple model for confinement in our calculations as we do not intend to present a detailed study of light meson spectroscopy. Our aim, as already stated in the previous section, is to study non leptonic decays of the B

meson to radially excited light meson states as well as to study the effects of radial mixings in the non leptonic decays $B \to \eta(\eta')K(K^*)$. We believe the conclusions reached on the basis of our calculations are likely to hold true in a more detailed model of confinement.

To begin with, we use the same harmonic confining potential as well as the other parameters used in Ref[2] to obtain the eigenstates and eigenvalues for the mass matrix in Eqn. 7. The various parameters used in the calculation are $m_u = m_d = 0.350$ GeV, $m_s = 0.503$ GeV, the angular frequency, ω =0.365 GeV and $b = B/m_u^2$ =0.09.

We obtain for the eigenvalues and eigenstates in the ρ system

$$\begin{aligned} |\rho(0.768)> &= 0.990|N>_0 +0.124|N>_1 -0.066|N>_2 \\ |\rho(1.545)> &= 0.108|N>_0 -0.973|N>_1 +0.204|N>_2 \\ |\rho(2.370)> &= 0.089|N>_0 -0.195|N>_1 +0.977|N>_2 \end{aligned} \tag{8}$$

To see how this result changes with a different confining potential we use a power law potential $V(r) = \lambda r^n$ [5] . We will use a linear and a quartic confining potential and compare the spectrum with that obtained with a harmonic oscillator potential. To fix the coefficient λ we require that the energy eigenvalues of the Schrödinger equation are similar in the least square sense with the energy eigenvalues used in Ref[2]. So for example, for the linear potential, we demand that

$$F = \sum_n (E_n(harmonic) - E_n(linear))^2$$

is a minimum. This fixes the constant λ in $V(r) = \lambda r$ and we obtain

$$\begin{aligned} |\rho(0.775)> &= 0.992|N>_0 +0.112|N>_1 -0.053|N>_2 \\ |\rho(1.515)> &= 0.104|N>_0 -0.986|N>_1 +0.130|N>_2 \\ |\rho(2.260)> &= 0.066|N>_0 -0.122|N>_1 +0.99|N>_2 \end{aligned} \tag{9}$$

We follow the same procedure for the quartic potential and obtain

$$\begin{aligned} |\rho(0.759)> &= 0.988|N>_0 +0.129|N>_1 -0.077|N>_2 \\ |\rho(1.567)> &= 0.103|N>_0 -0.955|N>_1 +0.278|N>_2 \\ |\rho(2.550)> &= 0.11|N>_0 -0.267|N>_1 +0.957|N>_2 \end{aligned} \tag{10}$$

We observe that the mass eigenstates and eigenvalues are not very sensitive to the confining potential and the radial mixing effects are small.

We next turn to mixing in the $\eta - \eta'$ system. In the traditional picture the η and η' mesons are mixtures of singlet and octet states η_1 and η_8 of $SU(3)$.

$$\begin{bmatrix} \eta \\ \eta' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \eta_8 \\ \eta_1 \end{bmatrix} \tag{11}$$

$$\eta_8 = \frac{1}{\sqrt{6}}\left[u\bar{u} + d\bar{d} - 2s\bar{s}\right] \tag{12}$$

$$\eta_1 = \frac{1}{\sqrt{3}}\left[u\bar{u} + d\bar{d} + s\bar{s}\right] \tag{13}$$

where the mixing angle θ lies between -10^0 and -20^0[10].

To obtain the eigenstates and eigenvalues in the $\eta - \eta'$ system we diagonalize the mass matrix

$$\begin{aligned} < q'_a\bar{q}'_b, n'|M|q_a\bar{q}_b, n > &= \delta_{aa'}\delta_{bb'}\delta_{nn'}(m_a + m_b + E_n) + \delta_{aa'}\delta_{bb'}\frac{B}{m_a m_b}\vec{s}_a\cdot\vec{s}_b\psi_n(0)\psi_{n'}(0) \\ &+ \delta_{ab}\delta_{a'b'}\frac{A}{m_a m_b}\psi_n(0)\psi_{n'}(0) \end{aligned} \tag{14}$$

This has a similar structure as the ρ system but now we have the additional annihilation contribution with strength A that causes flavor mixing. In our calculations we use the phase convention in Ref[2] where the wavefunctions at the origin in configuration space, which enter in the hyperfine and annihilation terms in the mass matrix, are positive(negative) for the even(odd) radial excitations.

We try to fit the values of A and B to the measured masses. The mass matrix is 6×6 matrix which we diagonalize to make predictions for 6 masses and mixings. However, for the sake of brevity we will only give the predictions for η and η' masses and wavefunctions. Several solutions that give acceptable values of the masses can be obtained. We choose solutions for the linear, quadratic and quartic confining potentials that make similar predictions for the $\eta(\eta')$ masses:

For the linear potential we obtain with $B = 0.065m_u^2$ and $A = 0.045m_u^2$

$$\begin{aligned} |\eta(0.544)\rangle &= 0.961|N>_0 - 0.198|N>_1 + 0.108|N>_2 \\ &- 0.150|S>_0 + 0.050|S>_1 - 0.032|S>_2 \\ |\eta'(0.924)\rangle &= 0.170|N>_0 + 0.0385|N>_1 - 0.0157|N>_2 \\ &+ 0.974|S>_0 - 0.126|S>_1 + 0.0486|S>_2 \end{aligned} \tag{15}$$

For the harmonic potential we obtain with $B = 0.065m_u^2$ and $A = 0.065m_u^2$

$$|\eta(0.547)\rangle = 0.913|N>_0 - 0.252|N>_1 + 0.154|N>_2$$

$$
\begin{aligned}
& - \quad 0.249|S>_0 +0.107|S>_1 -0.076|S>_2 \\
|\eta'(0.931)\rangle &= 0.316|N>_0 +0.109|N>_1 -0.049|N>_2 \\
& + \quad 0.925|S>_0 -0.148|S>_1 +0.088|S>_2 \qquad (16)
\end{aligned}
$$

Our results for the harmonic potential is similar to the results obtained in Ref[2] where a slightly different mass mixing matrix than the one used here has been used to obtain the $\eta - \eta'$ mixing.

Finally, for the quartic potential we obtain with $B = 0.065 m_u^2$ and $A = 0.11 m_u^2$

$$
\begin{aligned}
|\eta(0.547)> &= 0.764|N>_0 -0.287|N>_1 +0.198|N>_2 \\
& - \quad 0.441|S>_0 +0.248|S>_1 -0.196|S>_2 \\
|\eta'(0.940)> &= 0.623|N>_0 +0.350|N>_1 -0.177|N>_2 \\
& + \quad 0.658|S>_0 -0.134|S>_1 +0.087|S>_2 \qquad (17)
\end{aligned}
$$

It is clear that the eigenstates of $\eta(\eta')$ system are sensitive to the confining potential and there can be substantial radial mixing which can then affect the predictions for the decays $B \to \eta(\eta')K(K^*)$. We note that as we move from the linear to the quartic potential the $\eta(\eta')$ mixing deviates more significantly from the ideal mixing case. This may be understood from the fact that the fit to annihilation term, A, is the largest for the quartic potential which leads to the largest deviation from the ideal mixing case. A standard $\eta - \eta'$ mixing often used in the literature is given by [6, 7]

$$
\begin{aligned}
|\eta'\rangle_{std} &= \frac{1}{\sqrt{6}}\left[u\bar{u} + d\bar{d} + 2s\bar{s}\right] \\
|\eta\rangle_{std} &= \frac{1}{\sqrt{3}}\left[u\bar{u} + d\bar{d} - s\bar{s}\right] \qquad (18)
\end{aligned}
$$

We can then write the $\eta - \eta'$ states obtained with the various confining potential and keeping only the ground states, in terms of the the states defined above. For the linear potential we find

$$
\begin{aligned}
|\eta'\rangle &= 0.89\,|\eta'\rangle_{std} - 0.43\,|\eta\rangle_{std} \\
|\eta\rangle &= 0.87\,|\eta\rangle_{std} + 0.43\,|\eta'\rangle_{std} \qquad (19)
\end{aligned}
$$

For the harmonic potential

$$
\begin{aligned}
|\eta'\rangle &= 0.94\,|\eta'\rangle_{std} - 0.27\,|\eta\rangle_{std} \\
|\eta\rangle &= 0.89\,|\eta\rangle_{std} + 0.32\,|\eta'\rangle_{std} \qquad (20)
\end{aligned}
$$

and finally for the quartic potential one finds

$$\begin{aligned} |\eta'\rangle &= 0.90\,|\eta'\rangle_{std} + 0.13\,|\eta\rangle_{std} \\ |\eta\rangle &= 0.88\,|\eta\rangle_{std} + 0.08\,|\eta'\rangle_{std} \end{aligned} \tag{21}$$

This shows that all three confining potentials give mixings for the $\eta-\eta'$ that have substantial overlap with the standard mixing but the mixing with the quartic potential is closest to the standard mixing in the sense that here one has the smallest component of the $|\eta\rangle_{std}$ ($|\eta'\rangle_{std}$ in $|\eta'\rangle$ ($|\eta\rangle$).

We now turn to the $\omega-\phi$ system. As in the the $\eta-\eta'$ system we diagonalize the mass matrix in Eqn. 14. We use the same value for the hyperfine interaction as used for the ρ system. Again, for the sake of brevity, we only give the wavefunctions for the ground and the first excited states.

For the linear potential we obtain with $B = 0.09m_u^2$ and $A = 0.005m_u^2$

$$\begin{aligned} |\omega(0.782)\rangle &= 0.991|N>_0 +0.123|N>_1 -0.058|N>_2 \\ &- 0.014|S>_0 +0.004|S>_1 -0.002|S>_2 \\ |\phi(1.05)\rangle &= 0.012|N>_0 +0.011|N>_1 -0.004|N>_2 \\ &+ 0.997|S>_0 +0.071|S>_1 -0.034|S>_2 \\ |\omega(1.52)\rangle &= -0.113|N>_0 +0.982|N>_1 +0.144|N>_2 \\ &- 0.006|S>_0 -0.034|S>_1 +0.004|S>_2 \\ |\phi(1.66)\rangle &= 0.007|N>_0 -0.030|N>_1 -0.014|N>_2 \\ &+ 0.068|S>_0 -0.994|S>_1 -0.077|S>_2 \end{aligned} \tag{22}$$

For the harmonic potential we obtain with $B = 0.09m_u^2$ and $A = 0.015m_u^2$

$$\begin{aligned} |\omega(0.783)\rangle &= 0.984|N>_0 +0.154|N>_1 -0.081|N>_2 \\ &- 0.033|S>_0 +0.011|S>_1 -0.007|S>_2 \\ |\phi(1.05)\rangle &= 0.026|N>_0 +0.029|N>_1 -0.011|N>_2 \\ &+ 0.994|S>_0 +0.089|S>_1 -0.048|S>_2 \\ |\omega(1.57)\rangle &= -0.126|N>_0 +0.948|N>_1 +0.256|N>_2 \\ &- 0.008|S>_0 -0.139|S>_1 +0.010|S>_2 \\ |\phi(1.68)\rangle &= 0.025|N>_0 -0.118|N>_1 -0.07|N>_2 \\ &+ 0.082|S>_0 -0.976|S>_1 -0.143|S>_2 \end{aligned} \tag{23}$$

For the quartic potential we obtain with $B = 0.09m_u^2$ and $A = 0.023m_u^2$

$$\begin{aligned}
|\omega(0.782)\rangle &= 0.980|N>_0 +0.163|N>_1 -0.096|N>_2 \\
&- 0.049|S>_0 +0.012|S>_1 -0.009|S>_2 \\
|\phi(1.05)\rangle &= 0.041|N>_0 +0.034|N>_1 -0.015|N>_2 \\
&+ 0.991|S>_0 +0.100|S>_1 -0.060|S>_2 \\
|\omega(1.58)\rangle &= -0.122|N>_0 +0.932|N>_1 +0.322|N>_2 \\
&- 0.010|S>_0 -0.113|S>_1 +0.006|S>_2 \\
|\phi(1.7)\rangle &= 0.022|N>_0 -0.089|N>_1 -0.067|N>_2 \\
&+ 0.086|S>_0 -0.968|S>_1 -0.207|S>_2 \qquad (24)
\end{aligned}$$

As in the ρ system we find the mixing to be insensitive to the confining potential and we also find the effects of radial mixing to be small. We also find, as expected, a smaller value for the annihilation term in the fits to the masses as compared to the pseudoscalar system.

3 Effective Hamiltonian, Factorization and the Diagrammatic approach

In the Standard Model (SM) the amplitudes for hadronic B decays are generated by the following effective Hamiltonian [8]:

$$H_{eff}^q = \frac{G_F}{\sqrt{2}}[V_{fb}V_{fq}^*(c_1O_{1f}^q + c_2O_{2f}^q) - \sum_{i=3}^{10}(V_{ub}V_{uq}^*c_i^u + V_{cb}V_{cq}^*c_i^c + V_{tb}V_{tq}^*c_i^t)O_i^q] + H.C. \qquad (25)$$

where the superscript u, c, t indicates the internal quark, f can be u or c quark and q can be either a d or a s quark depending on whether the decay is a $\Delta S = 0$ or $\Delta S = -1$ process. The operators O_i^q are defined as

$$\begin{aligned}
O_{1f}^q &= \bar{q}_\alpha\gamma_\mu L f_\beta \bar{f}_\beta\gamma^\mu L b_\alpha \qquad O_{2f}^q = \bar{q}\gamma_\mu L f\bar{f}\gamma^\mu L b \\
O_{3,5}^q &= \bar{q}\gamma_\mu L b\bar{q}'\gamma_\mu L(R)q' \qquad O_{4,6}^q = \bar{q}_\alpha\gamma_\mu L b_\beta \bar{q}'_\beta\gamma_\mu L(R)q'_\alpha \qquad (26) \\
O_{7,9}^q &= \frac{3}{2}\bar{q}\gamma_\mu L b e_{q'}\bar{q}'\gamma^\mu R(L)q' \quad O_{8,10}^q = \frac{3}{2}\bar{q}_\alpha\gamma_\mu L b_\beta e_{q'}\bar{q}'_\beta\gamma_\mu R(L)q'_\alpha
\end{aligned}$$

where $R(L) = 1 \pm \gamma_5$, and q' is summed over u, d, and s. O_1 and O_2 are the tree level and QCD corrected operators. O_{3-6} are the strong gluon induced penguin operators, and operators O_{7-10} are due to γ and Z exchange (electroweak penguins), and "box" diagrams at loop level. The Wilson coefficients c_i^f are defined at the scale $\mu \approx m_b$ and have been evaluated to next-to-leading order in QCD. The c_i^t are the regularization scheme independent values obtained in Ref. [9]. We give the non-zero c_i^f below for $m_t = 176$ GeV, $\alpha_s(m_Z) = 0.117$, and $\mu = m_b = 5$ GeV,

$$\begin{aligned} c_1 &= -0.307\ ,\ \ c_2 = 1.147\ ,\ \ c_3^t = 0.017\ ,\ \ c_4^t = -0.037\ ,\ \ c_5^t = 0.010\ , c_6^t = -0.045\ , \\ c_7^t &= -1.24\times 10^{-5}\ ,\ \ c_8^t = 3.77\times 10^{-4}\ ,\ \ c_9^t = -0.010\ ,\ \ c_{10}^t = 2.06\times 10^{-3}\ , \\ c_{3,5}^{u,c} &= -c_{4,6}^{u,c}/N_c = P_s^{u,c}/N_c\ ,\ \ c_{7,9}^{u,c} = P_e^{u,c}\ ,\ \ c_{8,10}^{u,c} = 0 \end{aligned} \tag{27}$$

where N_c is the number of colors. The leading contributions to $P^i_{s,e}$ are given by: $P_s^i = (\frac{\alpha_s}{8\pi})c_2(\frac{10}{9}+G(m_i,\mu,q^2))$ and $P_e^i = (\frac{\alpha_{em}}{9\pi})(N_c c_1+c_2)(\frac{10}{9}+G(m_i,\mu,q^2))$. The function $G(m,\mu,q^2)$ is given by

$$G(m,\mu,q^2) = 4\int_0^1 x(1-x)\mathrm{ln}\frac{m^2-x(1-x)q^2}{\mu^2}\ \mathrm{d}x \tag{28}$$

All the above coefficients are obtained up to one loop order in electroweak interactions. The momentum q is the momentum carried by the virtual gluon in the penguin diagram. When $q^2 > 4m^2$, $G(m,\mu,q^2)$ develops an imaginary part. In our calculation, we use, for the current quark masses at the scale $\mu \sim m_b$, $m_u = 5$ MeV, $m_d = 7$ MeV, $m_s = 100$ MeV, $m_c = 1.35$ GeV [10, 11] and use $q^2 = m_b^2/2$.

The structure of the effective Hamiltonian allows us to write the amplitude for $B \to M_1M_2$ as

$$A \ = \ \frac{G_F}{\sqrt{2}}[V_{fb}V_{fq}^*T - (V_{ub}V_{uq}^*P_u + V_{cb}V_{cq}^*P_c + V_{tb}V_{tq}^*P_t] \tag{29}$$

Now we can write the tree amplitude T as

$$\begin{aligned} T &= \langle M_1, M_2|\, c_1O_{1f} + c_2O_{2f}\, |B\rangle \\ T &= c_2\, \langle M_1, M_2|\, O_{2f} - (1/N_1)O_{1f}\, |B\rangle \end{aligned} \tag{30}$$

where from Eqn. 27 $N_1 = -c_2/c_1 = 1.147/0.307 = 3.73$. In the factorization assumption there are two contributions to the tree matrix element, T. To be specific consider the decay $B^- \to \rho^0(\omega)\pi^-$. In this case there can be two contributions to T, given in the factorization assumption by

$$\begin{aligned} T &= T_1 + T_2 \\ T_1 &= c_2 < \pi^-|\bar{d}\gamma^\mu Lb|B^- >< \rho^0(\omega)|\bar{u}\gamma_\mu Lu|0> [\frac{1}{N_c} - \frac{1}{N_1}] \\ T_2 &= c_2 < \rho^0(\omega)|\bar{u}\gamma^\mu Lb|B^- >< \pi^-|\bar{d}\gamma_\mu Lu|0> [1 - \frac{1}{N_cN_1}] \end{aligned} \tag{31}$$

In the diagrammatic language the first term, T_2, corresponds to a b quark transition to a u quark and a W^- which turns into a π^-. The u quark then combines with the spectator quark to form the $\rho^0(\omega)$ particle. In the term T_1, the u quark from the $b \to u$ transition combines with the $\bar{u}$ quark from the W^- to form the $\rho^0(\omega)$ particle while the d quark from the W^- combines with the spectator quark to form the π^-. This is the color suppressed diagram and from the expression above we see that there is an additional suppression coming from the Wilson's coefficients and so, effectively T_1 is suppressed by a factor of $(1/N_c - 1/N_1) \sim 1/15$ relative to T_2.

We now turn to the penguin contribution, and for simplicity, we just concentrate on the the t penguin, P_t. We can write

$$P_t \;=\; \langle M_1, M_2| \sum_i c_i^t O_i \,|B\rangle \tag{32}$$

Again from the values of the Wilson's coefficients given in Eqn. 27 we can write

$$\begin{aligned} P_t &\approx \langle M_1, M_2| \, c_3^t O_3 + c_4^t O_4 \,|B\rangle + \langle M_1, M_2| \, c_5^t O_5 + c_6^t O_6 \,|B\rangle \\ &- \frac{1}{2} c_9^t \,\langle M_1, M_2| \, O_9 \,|B\rangle \\ P_t &\approx c_4^t \,\langle M_1, M_2| \, O_4 - (1/N_2) O_3 \,|B\rangle + c_6^t \,\langle M_1, M_2| \, O_6 - (1/N_3) O_5 \,|B\rangle \\ &- \frac{1}{2} c_9^t \,\langle M_1, M_2| \, O_9 \,|B\rangle \end{aligned} \tag{33}$$

where from Eqn. 27 $N_2 = -c_4^t/c_3^t = 0.037/0.017 = 2.2$ and $N_3 = -c_6^t/c_5^t = 0.045/0.010 = 4.5$.

In the diagrammatic approach the $\bar{q}'q'$ quarks appearing in the operator $O_3 - O_6$ appears from a "gluon" splitting into a $\bar{q}'q'$ pair while in the case the operators $O_7 - O_{10}$ it is a "γ", "Z" splitting into a $\bar{q}'q'$ pair.

Concentrating on only the term proportional to c_4^t, we can write in the factorization assumption,

$$\begin{aligned} P_t &= P_{t1} + P_{t2} \\ P_{t1} &= c_4^t (1 - \frac{1}{N_c N_2}) \Big[< \rho^0(\omega)|\bar{u}\gamma^\mu L b|B^- >< \pi^-|\bar{d}\gamma_\mu L u|0 > \\ &+ \; < \pi^-|\bar{d}\gamma^\mu L b|B^- >< \rho^0(\omega)|\bar{d}\gamma_\mu L d|0 > \Big] \\ P_{t2} &= c_4^t < \pi^-|\bar{d}\gamma^\mu L b|B^- >< \rho^0(\omega)|\bar{u}\gamma_\mu L u + \bar{d}\gamma_\mu L d|0 > [\frac{1}{N_c} - \frac{1}{N_2}] \end{aligned} \tag{34}$$

We see that the second term, P_{t2} has a suppression factor of $(1/N_c - 1/N_2) \sim 1/8$. This term is called OZI suppressed and in the diagrammatic language this is shown as a "gluon" splitting into a quark-antiquark pair which then hadronizes to a hadron. Of course for a real gluon this process is forbidden as the color octet gluon cannot form a color singlet hadron. Note that there can be other OZI violating diagrams that have been considered to explain the large branching ratios in the decay $B \to \eta' K$ and the semi-inclusive decay $B \to \eta' X_s$. In these diagrams the enhanced branching ratios are due to the anomaly, gluon couplings to the flavour singlet component of the η' or the intrinsic charm content of the η' [12, 13]. We will not consider such diagrams in our analysis.

The terms represented by P_{t1}, in the diagrammatic approach, has a "gluon" splitting into a quark- antiquark pair but now the antiquark combines with the d quark, coming from the $b \to d$ transition, to form a meson while the other quark combines with the spectator quark to form the second meson in the final state. The two terms represented in P_{t1} represent the cases where the quark-antiquark pair from the "gluon" is a $\bar{u}u$ and a $\bar{d}d$ pair.

One can do the same exercise with the term proportional to c_6^t in Eqn. 33 and in this case the OZI violating term is suppressed by $(1/N_c - 1/N_3) \sim 1/9$. Note the term P_{t2} from the electroweak penguin term c_9^t does not have any suppression. This is expected as a $\bar{q}q$ pair from a γ or Z boson is in a color singlet state and therefore can form a hadron.

For the case of $B^- \to \eta(\eta')K$ the term P_{t1} and P_{t2} are similar to the one above.

$$
\begin{aligned}
P_t &= P_{t1} + P_{t2} \\
P_{t1} &= c_4^t (1 - \frac{1}{N_c N_2}) \Big[< \eta(\eta')|\bar{u}\gamma^\mu L b|B^- >< K^-|\bar{s}\gamma_\mu L u|0 > \\
&+ \; < K^-|\bar{s}\gamma^\mu L b|B^- >< \eta(\eta')|\bar{s}\gamma_\mu L s|0 > \Big] \\
P_{t2} &= c_4^t < K^-|\bar{s}\gamma^\mu L b|B^- >< \eta(\eta')|\bar{u}\gamma_\mu L u + \bar{d}\gamma_\mu L d + \bar{s}\gamma_\mu L s|0 > [\frac{1}{N_c} - \frac{1}{N_2}]
\end{aligned} \tag{35}
$$

Note we again find the OZI term to be suppressed by a factor $\sim 1/8$. However if terms in P_{t1} interfere destructively then the OZI terms may become important. Note the suppression in the OZI term can also be diluted if the contributions from the u,d and s term interfere constructively in P_{t2}.

In the decay $B^- \to \rho^0(\omega)\pi^-$ the OZI suppressed term does not play an important role, as these decays are not penguin dominated because the CKM factors in the tree and penguin terms are of the same order and the penguins are loop suppressed. This fact is also supported by recent experimental measurement of the branching ratios $BR[B^- \to \omega\pi^-] \sim 11.3 \times 10^{-6}$ and $BR[B^- \to \rho^0\pi^-] \sim 10.4 \times 10^{-6}$ [14]. Note that in Eqn. 34, from the flavor structure of the ρ^0 and ω wavefunction, it is obvious that the two terms in in P_{t1} interfere destructively for the ρ^0 but constructively for the ω. Furthermore the OZI suppressed term, P_{t2} for the ρ^0 vanishes, neglecting the electroweak contribution, but not for the ω. This means that the penguin term for $B^- \to \rho\pi^-$ is smaller than in $B^- \to \omega\pi^-$. Hence if the penguin terms were dominant then there would be a significant difference between the branching ratios $BR[B^- \to \omega\pi^-]$ and $BR[B^- \to \rho\pi^-]$. Hence the small measured difference in the branching ratios for $B^- \to \rho^0\pi^-$ and $B^- \to \omega\pi^-$ implies relatively small penguin effects in these decays.

However decays of the type $B \to \eta K$ are dominated by penguin terms because of large CKM factors in the penguin terms compared to the tree term. Here, the OZI suppressed terms may have significant effects on the predictions of these decays. Note that we expect the OZI suppressed terms to be more important in $B \to KP$ than in $B \to KV$ decays where P is a pseudoscalar and V is a vector state. This follows from the fact that in J/ψ and Υ decays we know that the OZI-forbidden process requires three gluons for coupling to a vector meson and two gluons for coupling to a pseudoscalar. Thus one would expect that the contribution of the OZI suppressed term should be much smaller in the $B \to K\rho^0(\omega)$ and $B \to K\phi$ decays than in $B \to K\eta$ and $B \to K\eta'$ decays [15, 16]. One of the authors of this work has shown that one can make definite predictions about the branching ratios $B \to \eta K/B \to \eta' K$ and $B \to \eta' K^*/B \to \eta K^*$ [17, 18] if one assumes that the OZI terms are forbidden. We will first derive these predictions in the language of effective Hamiltonian and using the factorization assumption. We then study how the predictions change if we use non standard mixing in the $\eta - \eta'$ sector and if we include the OZI terms.

4 $B \to \rho, \omega, \phi$ transitions.

In this section we study decays of the type $B \to VP$ where $V = \rho, \omega, \phi$. As we found in section 2 the wavefunction of V has the general form,

$$|V\rangle \;=\; \sum_i a_i \, |N_i\rangle \tag{36}$$

We can then write,

$$Amp(B \to VM) \;=\; \sum_i a_i \, \langle M, N_i| \, H_{eff} \, |B\rangle \tag{37}$$

We now consider the ratios of the following decays

$$R_{\rho^+} \;=\; BR(\bar{B}^0 \to \pi^- \rho^{+\prime})/BR(\bar{B}^0 \to \pi^- \rho^+) \tag{38}$$

$$R_{\rho^0} = BR(\bar{B}^- \to \pi^- \rho^{0\prime})/BR(\bar{B}^- \to \pi^- \rho^0) \tag{39}$$

$$R_\omega = BR(B^- \to \pi^- \omega')/BR(B^- \to \pi^- \omega) \tag{40}$$

$$R_\phi = BR(B_s \to \pi^0 \phi')/BR(B_s \to \pi^0 \phi) \tag{41}$$

As discussed in the previous section we can neglect the penguin contribution to these decays. Note that for the decay $B_s \to \pi^0 \phi$ there is no contribution from the QCD penguin and the dominant electroweak penguin term has the same structure as the tree amplitude and hence the ratio R_ϕ remains essentially the same even in the presence of penguin terms. We then obtain

$$\begin{aligned} R_{\rho^+} &= \left| \frac{\langle \rho^{+\prime}| \bar{u}\gamma^\mu(1-\gamma_5)b \left|\bar{B}^0\right\rangle \langle \pi^-| \bar{d}\gamma_\mu(1-\gamma_5)u \left|0\right\rangle}{\langle \rho^{+}| \bar{u}\gamma^\mu(1-\gamma_5)b \left|\bar{B}^0\right\rangle \langle \pi^-| \bar{d}\gamma_\mu(1-\gamma_5)u \left|0\right\rangle} \right|^2 \\ &= \left|\frac{A_0'}{A_0}\right|^2 \frac{P^3_{\rho^{+\prime}}}{P^3_\rho} \end{aligned} \tag{42}$$

where P is the magnitude of the three momentum of the final states and the form factor A^0 is defined through

$$\begin{aligned} \langle V_f| A_\mu \left|P_i\right\rangle &= (M_i + M_f)A_1 \left[\epsilon^*_\mu - \frac{\epsilon^*.q}{q^2} q_\mu\right] - A_2 \frac{\epsilon^*.q}{M_i + M_f}\left[(P_i+P_f)_\mu - \frac{M_i^2 - M_f^2}{q^2} q_\mu\right] \\ &+ 2M_f A_0 \frac{\epsilon^*.q}{q^2} q_\mu \end{aligned} \tag{43}$$

$$\begin{aligned} R_{\rho^0} &\approx \left| \frac{\langle \rho^{0\prime}| \bar{u}\gamma^\mu(1-\gamma_5)b \left|\bar{B}^0\right\rangle \langle \pi^-| \bar{d}\gamma_\mu(1-\gamma_5)u \left|0\right\rangle}{\langle \rho^{0}| \bar{u}\gamma^\mu(1-\gamma_5)b \left|\bar{B}^0\right\rangle \langle \pi^-| \bar{d}\gamma_\mu(1-\gamma_5)u \left|0\right\rangle} \right|^2 \\ &= \left|\frac{A_0'}{A_0}\right|^2 \frac{P^3_{\rho^{0\prime}}}{P^3_{\rho^0}} \end{aligned} \tag{44}$$

In the above we have ignored the term in the amplitude for $B^- \to \rho(\rho')\pi^-$ which is given by T_1 in Eqn. 31 and hence is suppressed by $\sim 1/15$ relative to the dominant term T_2.

Within the same approximation we can then write

$$\begin{aligned} R_{\omega} &\approx \left| \frac{\langle \omega'| \bar{u}\gamma^\mu(1-\gamma_5)b \left|\bar{B}^0\right\rangle \langle \pi^-| \bar{d}\gamma_\mu(1-\gamma_5)u \left|0\right\rangle}{\langle \omega| \bar{u}\gamma^\mu(1-\gamma_5)b \left|\bar{B}^0\right\rangle \langle \pi^-| \bar{d}\gamma_\mu(1-\gamma_5)u \left|0\right\rangle} \right|^2 \\ &= \left|\frac{A_0'}{A_0}\right|^2 \frac{P^3_{\omega'}}{P^3_{\omega}} \end{aligned} \tag{45}$$

Finally,

$$
\begin{aligned}
R_\phi &= \left| \frac{\langle \phi\prime | \, \bar{s}\gamma^\mu (1-\gamma_5) b \left| \bar{B}_s \right\rangle \langle \pi^0 | \, \bar{u}\gamma_\mu (1-\gamma_5) u \, |0\rangle}{\langle \phi | \, \bar{s}\gamma^\mu (1-\gamma_5) b \left| \bar{B}^0 \right\rangle \langle \pi^0 | \, \bar{u}\gamma_\mu (1-\gamma_5) u \, |0\rangle} \right|^2 \\
&= \left| \frac{A_0'}{A_0} \right|^2 \frac{P_{\phi\prime}^3}{P_\phi^3} \qquad (46)
\end{aligned}
$$

To calculate the ratios we need the form factor A_0. We use the model for form factors in Ref[19] that incorporates some relativistic features and is relatively simple to use. The calculation of the form factors in this model is not always in agreement with experimental results, however, we expect the model to make reliable predictions for the ratio of form factors. We believe this feature to be true for most other models for form factors. Hence we shall mostly calculate quantities that can be expressed as ratios of form factors. The model in [19] assumes, like many other models, the weak binding limit which sets the meson mass to be the sum of the masses of the constituent quarks making up the meson. Hence the effect of binding are included in the quark masses. This is a good approximation for the lowest resonances of the ρ, ω and the ϕ systems. In this model the form factor A_0 is given by

$$
\begin{aligned}
A_0 &= A[J_1 - \frac{M_-}{M_+} J_2] \\
A &= \frac{\sqrt{4 M_i M_f} M_+}{M_+^2 - q^2} \\
J_1 &= \int d^3 p \phi_f^*(\vec{p} + \vec{a}) \phi_i(\vec{p}) \\
J_2 &= m_s \int d^3 p \phi_f^*(\vec{p} + \vec{a}) \phi_i(\vec{p}) [\frac{\vec{p}.\vec{a}}{\mu a^2} + \frac{1}{m_f}]
\end{aligned}
$$

where

$$
\begin{aligned}
M_\pm &= M_f \pm M_i \\
\vec{a} &= 2 m_s \vec{\beta} = 2 \frac{m_s \tilde{q}}{M_+} \\
\tilde{q}^2 &= M_+^2 \frac{M_-^2 - q^2}{M_+^2 - q^2} \\
\mu &= \frac{m_i m_f}{m_i + m_f} \qquad (47)
\end{aligned}
$$

and ϕ_f and ϕ_i represent the momentum space wave functions while $\vec{\beta}$ is the velocity of the mesons in the equal velocity frame and $m_{i,f}$ are the non spectator quark masses of the initial and final meson.

We use the momentum wavefunction ϕ_f obtained from spectroscopy in section. (2) while for ϕ_i we use the wave function

$$\phi_i = \phi_B = N_B e^{-p^2/p_F^2} \tag{48}$$

where p_F is the Fermi momentum of the B meson. In our calculations we will take $p_F =$ 300MeV. Note that in the analysis presented in the introduction we have neglected the Fermi momentum of the b quark, but since p_F/m_b is small the general conclusions reached in the introduction still continue to be valid.

For transitions to higher resonant states, we use the same quark masses as those used in the transition of the B meson to the lowest resonant state. This is reasonable, as the spectator quark still comes from the B meson and therefore has the same value for its mass irrespective of whether the final state is in the lowest or the first excited state. The values for the masses of the non spectator masses are taken to be essentially the same as those used in section. (2) for spectroscopy. However for the calculation of the velocity $\vec{\beta}$ and hence $\vec{a}$ defined in Eqn. 47 we use the physical mass of the higher resonant state.

Table 1: Ratios of branching ratios for different confining potentials

Ratio	Linear	Quadratic	Quartic
R_{ρ^+}	2.3	2.0	1.9
R_{ρ^0}	2.3	2.0	1.9
R_ω	3.5	2.5	1.7
R_ϕ	6.7	6.2	5.2

In Table. 1 we give our predictions for the various ratios defined above. We find that the transitions to higher excited states can be comparable or enhanced relative to the transitions to the ground state. From Table. 1 we see that the ratios of branching ratios are slightly sensitive to the confining potential and the ratios of branching ratios increase as we go from the quartic to the linear potential. This is because the wavefunction for the linear potential has a longer tail and hence more high momentum components than the wavefunction for the quadratic and the quartic potentials. The wavefunction for the quadratic potential, has in turn, a longer tail and hence more high momentum components than the wavefunction for the quartic potential. As mentioned in section 1 the form factor in $B \to M$ transition, where M is a light meson, is sensitive to the high momentum tail of the meson wavefunction. Hence we would expect the hierarchy $(A_{1,0})_{linear} > (A_{1,0})_{quadratic} > (A_{1,0})_{quartic}$ where $A_{1,0}$

are the form factors for the transition of B to the first radially excited and the ground state of the meson M. We see from from Table. 1 that this hierarchy is maintained for the ratios of form factors and so we have $(A_1/A_0)_{linear} > (A_1/A_0)_{quadratic} > (A_1/A_0)_{quartic}$. Note that the ratio of form factors also depend on the choice of the Fermi momentum of the B meson, as a smaller(larger) Fermi momentum would make the form factors more(less) sensitive to the tail of the wavefunction of M, as well as mixing effects in the wavefunction of the meson M.

One can check that the predictions for the ratios of branching ratios are not very different from the case where we neglect radial mixings. Hence one concludes that the effects of radial mixing are in most cases small and negligible for practical applications. We observe in Table. 1 that there can be a large enhancement for R_ϕ. One can get a rough estimate of the branching ratio for $B_s \to \phi\pi^0$ from

$$\frac{BR[B_s \to \phi\pi^0]}{BR[\bar{B}^0 \to \rho^+\pi^-]} \approx \frac{1}{2}\left|\frac{V_{ub}V_{us}^*(c1 + c2/N_c) - V_{tb}V_{ts}^* c_9/2}{V_{ub}V_{ud}^*(c2 + c1/N_c)}\right| \approx 0.04$$

where we have neglected form factor and phase space differences between $B_s \to \phi\pi^0$ and $\bar{B}^0 \to \rho^+\pi^-$. Using the measured $BR[B \to \rho^+\pi^-] \sim 28 \times 10^{-6}$ [14] we get $BR[B_s \to \phi\pi^0] \sim 10^{-6}$. Hence the large enhancement for R_ϕ indicates that $BR[B_s \to \phi'\pi^0]$ can be of $O(10^{-5})$.

Note that in the $\rho(\omega)$ system there are two resonances, $\rho(1450)[\omega(1420)]$ and $\rho(1700)[\omega[1650]$, which can be identified a S-wave radial excitation(2S) and a D wave orbital excitation in the quark model. However recent studies of the decays of these resonances show that it is possible that these states are mixtures of $q\bar{q}$ and hybrid states Ref[10]. Hence the state $\rho(1450)[\omega(1420)]$ is interpreted as a 2S state with a small mixture of a hybrid state. We do not take into account such possible mixing with a hybrid state in our calculation and the meson masses for these excited states used in our calculation are the ones we predict in section 2. For the ϕ system there is only state at $\phi(1680)$ which we interpret as a 2S state in the absence of mixing effects.

5 $B^\pm \to K^\pm(K^{*\pm})\eta'(\eta)$

We construct two ratios

$$R_K = BR(B^- \to K^-\eta)/BR(B \to K^-\eta') \tag{49}$$

and

$$R_{K^*} = BR(B^- \to K^{-*}\eta')/BR(B \to K^{-*}\eta) \tag{50}$$

Let us assume the $\eta - \eta'$ mixing used in Ref[17]

$$
\begin{aligned}
|\eta\rangle &= \frac{1}{\sqrt{2}}\left[N_0 - S_0\right] \\
|\eta'\rangle &= \frac{1}{\sqrt{2}}\left[N_0 + S_0\right]
\end{aligned} \tag{51}
$$

where, as before, $|N >= |u\bar{u} + d\bar{d} > /\sqrt{2}$ and $S = |s\bar{s} >$. Now, from Eqn. 35 if we only include the term P_{t1} then we find

$$
R_K \approx \left|\frac{f_K F_\eta^+ + f_\eta^s F_K^+}{f_k F_{\eta'}^+ + f_{\eta'}^s F_K^+}\right|^2 \tag{52}
$$

where we have dropped the masses of the pseudoscalars in the final states and the form factor F^+ is defined through

$$
\begin{aligned}
\langle P(p_f)|\,\bar{q}\gamma_\mu(1-\gamma_5)b\,|B(p_i)\rangle &= \left[(p_i+p_f)_\mu - \frac{m_B^2 - m_P^2}{q^2}q_\mu\right]F_1(q^2) \\
&+ \frac{m_B^2 - m_P^2}{q^2}q_\mu F_0(q^2) \\
&= F_P^+(p_i+p_f)_\mu + F_P^- q_\mu .
\end{aligned} \tag{53}
$$

In the above equation f_K is the kaon decay constant and the decay constants f_η^q and $f_{\eta'}^q$ are defined by,

$$
i f^q_{\eta(\eta')} p^\mu_{\eta(\eta')} = \langle \eta(\eta')|\,\bar{q}\gamma^\mu(1-\gamma_5)q\,|0\rangle . \tag{54}
$$

For the mixing in Eqn. 51, and assuming $SU(3)$ flavor symmetry, we can write

$$
\begin{aligned}
f_\eta^{u,d} &\approx \frac{f_K}{2} \\
f_\eta^s &\approx \frac{-f_K}{\sqrt{2}} \\
f_{\eta'}^{u,d} &\approx \frac{f_K}{2} \\
f_{\eta'}^s &\approx \frac{f_K}{\sqrt{2}} \\
F_\eta^+ &\approx \frac{F_K^+}{2} \\
F_{\eta'}^+ &\approx \frac{F_K^+}{2}
\end{aligned} \tag{55}
$$

One can then write

$$R_K \approx \left|\frac{\frac{1}{2}-\frac{1}{\sqrt{2}}}{\frac{1}{2}+\frac{1}{\sqrt{2}}}\right|^2 \sim 0.03 \tag{56}$$

It was shown in Ref[17, 18] that there is a parity selection rule in the decays $B \to \eta(\eta')K^{(*)}$ which fixes the relative phase between the penguin amplitudes representing the strange and nonstrange contributions to η and η' final states. In particular the parity selection rule predicts that the phases between the strange and nonstrange penguin amplitudes in $B \to K\eta(\eta')$ and $B \to K^*\eta(\eta')$ are reversed. Hence neglecting form factor differences for $B \to P$ and $B \to V$ transitions, one obtains

$$R_{K^*} \approx \left|\frac{\frac{1}{2}-\frac{1}{\sqrt{2}}}{\frac{1}{2}+\frac{1}{\sqrt{2}}}\right|^2 \sim 0.03 \tag{57}$$

We now calculate the ratios above with a nonstandard $\eta - \eta'$ mixing. We will use the mixing for a quartic potential given in Eqn. 17. This is because the ground states of this mixing has the largest overlap with the standard mixing in Eqn. 51. Later on in this paper we will compare form factors calculated with the mixing in Eqn. 17 with those calculated with the mixing in Eqn. 51 to get an idea of the effects of radial mixing in the $\eta - \eta'$ wavefunctions. A key ingredient in the parity selection rule that predicts the relative phases of the strange and nonstrange contributions to η and η' final states is approximate flavor symmetry. In the flavor symmetry limit the radial wave functions of π, K, η and η' (up to mixing factors in $\eta(\eta')$ are all the same. One can then argue that since all are ground state wave functions with no nodes and a constant phase over the entire radial domain, flavor symmetry breaking can change the radial shape and size of the wave function but will not reverse the phases. This is probably a reasonable assumption as long as we only have ground state wave functions. But when radially excited wave functions which have nodes are also considered, there is a phase reversal in the wave function. Note that with factorization, the B decay into the nonstrange part of the η and η' involves a point like form factor for the kaon and and a hadronic overlap integral for the $\eta(\eta')$. But the B decay into the strange part of the η and η' involves a point like form factor for the $\eta(\eta')$ and a hadronic overlap integral for the kaon. The penguin amplitude that involves a hadronic overlap integral for the $\eta(\eta')$ can cause a phase ambiguity between the two penguin amplitudes because the wave function for the radial excitation changes phase and we are not able to exactly calculate flavor breaking effects.

A simple model to simulate flavor symmetry breaking would be to define, for the $\eta(\eta')$, an effective wavefunction

$$\Psi_{eff} = a_0\Psi_{N_0} + a_1 r_1 \Psi_{N_1} + a_2 r_2 \Psi_{N_2} \tag{58}$$

where the a_i's are from Eqn. 17 and r_1 and r_2, representing flavor breaking effects, can have both signs. The effective wavefunction Ψ_{eff} will then be used for form factor calculations. We will consider two choices of r_1 and r_2. In the first case we set $r_1 = r_2 = 1$ and in the second case we choose r_1 and r_2 to be such that the contributions from the various radial excitations add constructively in the form factor calculations involving the η and the η'.

The form factors in Eqn. 53 calculated using the quark model of Ref[19] are given by

$$\begin{aligned}
F_+ &= A[J_1 + \frac{M_-}{M_+}J_2] \\
F_- &= A[J_2 + \frac{M_-}{M_+}J_1] \\
A &= \frac{\sqrt{4M_iM_f}M_+}{M_+^2 - q^2} \\
J_1 &= \int d^3p\phi_f^*(\vec{p}+\vec{a})\phi_i(\vec{p}) \\
J_2 &= m_s\int d^3p\phi_f^*(\vec{p}+\vec{a})\phi_i(\vec{p})[\frac{\vec{p}.\vec{a}}{\mu a^2} + \frac{1}{m_f}]
\end{aligned} \tag{59}$$

where

$$\begin{aligned}
M_\pm &= M_i \pm M_f \\
\vec{a} &= 2m_s\vec{\beta} = 2\frac{m_s\tilde{q}}{M_+} \\
\tilde{q}^2 &= M_+^2\frac{M_-^2 - q^2}{M_+^2 - q^2} \\
\mu &= \frac{m_im_f}{m_i + m_f}
\end{aligned} \tag{60}$$

and ϕ_f and ϕ_i represent the momentum space wave functions, $\vec{\beta}$ is the velocity of the mesons in the equal velocity frame and $m_{i,f}$ are the non spectator quark masses of the initial and final meson. For the calculation of the form factors in the $\eta(\eta')$ system, clearly, the weak binding assumption, which would imply $M_\eta \sim M_{\eta'} \sim M_\rho$, does not hold. One does not know how to incorporate corrections to the weak binding limit. One could choose, for instance, different quark masses, and there are various reasonable choices of quark masses that can be made leading to very different predictions for the form factors. We do not wish to explore all these possibilities here, instead we will use the same procedure and the same values for the quark masses used in the calculation of the form factors in the vector system. We then find

$$\begin{aligned} \frac{F^{\eta'}_{+nonstandard}}{F^{\eta'}_{+standard}} &\approx 1.5, 1.7 \\ \frac{F^{\eta}_{+nonstandard}}{F^{\eta}_{+standard}} &\approx 0.5, 2.1 \end{aligned} \tag{61}$$

where $F_{+standard}$ and $F_{+nonstandard}$ are the form factors calculated in the standard mixing in Eqn. 51, and for the nonstandard mixing in Eqn. 17. The contribution from the form factor F_{-} is negligible. The two numbers in the equation above correspond to the two choices for $r_{1,2}$ mentioned above. We note that the form factor with nonstandard mixing does not change much for the η' but changes significantly for the η for the two choices of $r_{1,2}$.

Using the mixing in Eqn. 17 one obtains for the decay constants

$$\begin{aligned} \frac{f^{u,d}_{\eta_{nonstandard}}}{f^{u,d}_{\eta_{standard}}} &\approx 1 \\ \frac{f^{u,d}_{\eta'_{nonstandard}}}{f^{u,d}_{\eta'_{standard}}} &\approx 1.1 \\ \frac{f^{s}_{\eta_{nonstandard}}}{f^{s}_{\eta_{standard}}} &\approx 0.8 \\ \frac{f^{s}_{\eta'_{nonstandard}}}{f^{s}_{\eta'_{standard}}} &\approx 1.2 \end{aligned} \tag{62}$$

We will choose the second entry in Eqn. 61 and for simplicity use

$$\begin{aligned} \frac{F^{\eta'}_{+nonstandard}}{F^{\eta'}_{+standard}} &\approx 2.0 \approx \frac{F^{\eta}_{+nonstandard}}{F^{\eta}_{+standard}} \\ f^{u,d,s}_{\eta_{nonstandard}} &\approx f^{u,d,s}_{\eta_{standard}} \\ f^{u,d,s}_{\eta'_{nonstandard}} &\approx f^{u,d,s}_{\eta'_{standard}} \end{aligned} \tag{63}$$

The relations in Eqn. 63 are within the uncertainties in the calculation of the form factors in Eqn. 61 and Eqn. 62

It is now easy to check that the predictions for R_K and R_{K^*} in Eqn. 56 and Eqn. 57 remain essentially unchanged. However, as was shown by one of the authors of this paper, one can use flavour topology characteristics of charmless B decays to derive additional sum rules connecting B decays to $K\eta(\eta')$ and $K\pi$ final states. One of the interesting sum rule derived in [7, 16, 20] is, neglecting phase space corrections,

$$R = \frac{\Gamma[B^\pm \to K^\pm\eta'] + \Gamma[B^\pm \to K^\pm\eta]}{\Gamma[B^\pm \to K^\pm\pi^0]} \leq 3 \tag{64}$$

Note that this sum rule is true for any standard $\eta-\eta'$ mixing. Recent experimental measurements [21], however, show the above sum rule to be invalid. In the factorization assumption, one can write, neglecting the tree contribution,

$$R \approx \frac{\left|f_K F^+_\eta + f^s_\eta F^+_K\right|^2 + \left|f_K F^+_{\eta'} + f^s_{\eta'} F^+_K\right|^2}{\left|F^+_{\pi^0} f_K\right|^2}. \tag{65}$$

With the mixing in Eqn. 51 we get $R \approx 3$. Note that one can check that for any standard $\eta - \eta'$ mixing one always gets $R \approx 3$. With the nonstandard $\eta - \eta'$ mixing in Eqn. 17, we get $R \approx 6$ which is now consistent with experiment. One would also get similar predictions with a K^* in the final state. In particular we have

$$\begin{aligned}
\frac{\Gamma[B^\pm \to K^{*\pm}\eta]}{\Gamma[B^\pm \to K^{*\pm}\pi^0]} &= |\sqrt{2}+1|^2 \approx 6 \\
\frac{\Gamma[B^\pm \to K^{*\pm}\eta']}{\Gamma[B^\pm \to K^{*\pm}\pi^0]} &= |\sqrt{2}-1|^2 \approx \frac{1}{6}
\end{aligned} \tag{66}$$

As we have argued before the OZI suppressed terms may play an important role in the decays $B \to \eta(\eta')K^{(*)}$ decays. In particular the OZI suppressed terms are important for decays with a η' in the final state because they add constructively while for the η in the final state the OZI suppressed terms tend to cancel among themselves. One can calculate the contribution to the amplitude of the OZI suppressed term with the mixing in Eqn. 51 as,

$$\begin{aligned}
x_{\eta'} &\approx \frac{1}{8}(1+\frac{1}{\sqrt{2}}) \sim 0.21 \\
x_\eta &\approx \frac{1}{8}(1-\frac{1}{\sqrt{2}}) \sim 0.037
\end{aligned} \tag{67}$$

where we have dropped factors common to both x_η and $x_{\eta'}$. We see that the OZI suppressed contribution for the η' in the final state is indeed more important than for the η in the final state.

We now present the full amplitude for the decays $B \to \eta(\eta')K(K^*)$ including all the terms in the effective Hamiltonian in the factorization assumption and including the OZI suppressed terms. To make definite predictions we choose the form factors, $F^{\eta(\eta')}_{+standard}$, as well as the form factors for $B \to K(K^*)$ transitions to be given by Ref[22]. We will use the decay constants $f^{u,d}_{\eta} = f^{u,d}_{\eta'} \approx 0.8 f_\pi$ and $f^s_{\eta'} = -f^s_{\eta} \approx 1.3 f_\pi$.

Finally, one can write the amplitude for $B^- \to K^- \eta'$ as [23]

$$M = \frac{G_F}{\sqrt{2}}\left[V_u\left(a_1 r_1 Q_K + a_2 Q_{\eta'}\right) - \sum_{i=u,c,t} V_i\left\{(T_1^i r_1 + T_2^i r_2) Q_K + T_3^i Q_{\eta'}\right\}\right]$$

where

$$\begin{aligned} T_1^i &= 2a_3^i - 2a_5^i - \frac{1}{2}a_7^i + \frac{1}{2}a_9^i \\ T_2^i &= a_3^i + a_4^i - a_5^i + (2a_6^i - a_8^i)\frac{m^2_{\eta'}}{2m_s(m_b - m_s)} + \frac{1}{2}a_7^i - \frac{1}{2}a_9^i - \frac{1}{2}a_{10}^i \\ T_3^i &= a_4^i + 2(a_6^i + a_8^i)\frac{m_K^2}{m_u + m_s}\frac{1}{m_b - m_u} + a_{10}^i \end{aligned} \tag{68}$$

with $a_1 = c_1 + \frac{c_2}{N_c}$, $a_2 = c_2 + \frac{c_1}{N_c}$, $a^i_j = c^i_j + \frac{c^i_{j+1}}{N_c}$, $a^i_{j+1} = c^i_{j+1} + \frac{c^i_j}{N_c}$, $r_1 = \frac{f^u_{\eta'}}{f_\pi}$ $r_2 = \frac{f^s_{\eta'}}{f_\pi}$, $Q_K = iF_0^K(m^2_{\eta'})(m_B^2 - m_K^2) f_\pi$, $Q_{\eta'} = iF_0^{\eta'}(m_K^2)(m_B^2 - m^2_{\eta'}) f_K$, $V_i = V_u, V_c, V_t$ and N_c is effective number of colors.

In the above equations we have used the quark equations of motion to simplify certain matrix elements. The masses used in these equations of motion are the current quark masses given in section 3. The expression for the amplitude can also be used for $B \to \eta K$ by making the necessary changes. It is also straight forward to write down the amplitudes for $B \to K^*\eta'$ and $B \to K^*\eta$ decays.

$$M = \frac{G_F}{\sqrt{2}}\left[V_u\left(a_1 f^u_{\eta'} A + a_2 m_{K^*} g_{K^*} B\right) - \sum_{i=u,c,t} V_i\left\{(S_1^i f^u_{\eta'} + S_2^i f^s_{\eta'}) A + S_3^i m_{K^*} g_{K^*} B\right\}\right]$$

where

$$\begin{aligned} S_1^i &= 2a_3^i - 2a_5^i - \frac{1}{2}a_7^i + \frac{1}{2}a_9^i \\ S_2^i &= a_3^i + a_4^i - a_5^i - (2a_6^i - a_8^i)\frac{m^2_{\eta'}}{2m_s(m_b + m_s)} + \frac{1}{2}a_7^i - \frac{1}{2}a_9^i - \frac{1}{2}a_{10}^i \\ S_3^i &= a_4^i + a_{10}^i \end{aligned} \tag{69}$$

with $A = 2m_{K^*} A_0 \varepsilon^* \cdot p_B$, $B = 2\varepsilon^* \cdot p_B F_1^{\eta'}(m^2_{K^*})$ and we will use $g_{K^*} = 221$ MeV where g_{K^*} is the vector meson decay constant. A similar expression for $B \to \eta K^*$ can also be obtained. To identify the various terms in Eqn. 68 and Eqn. 69 , let us for simplicity drop the electroweak penguin terms represented by $a_7^i - a_{10}^i$. Note that the term proportional to a_6^i is formally of $O(1/m_b)$ and so strictly in the $m_b \to \infty$ limit this term vanishes. However for realistic quark masses this term is not negligible and is chirally enhanced because of the strange quark mass, m_s which is given in section 3. The OZI suppressed terms are represented by the terms a_3^i and a_5^i. One can check that if we drop the OZI suppressed terms as well as the chirally enhanced terms then, neglecting form factor and phase space differences, we would recover the predictions of Eqn. 56 and Eqn. 57 from Eqn. 68 and Eqn. 69. If we include the OZI suppressed terms in the calculation without the chirally enhanced contribution we obtain $BR[B \to \eta' K^*] = 1.1 \times 10^{-6}$ while if we ignore the OZI suppressed terms then we obtain $BR[B \to \eta' K^*] = 0.34 \times 10^{-6}$. Hence we see that the presence of the OZI suppressed term can alter significantly the $BR[B \to \eta' K^*]$. There is a much smaller effect of the OZI suppressed terms in $BR[B \to \eta' K]$. This, as already mentioned, is due to the fact that the OZI allowed terms in $BR[B \to \eta' K]$ tend to add constructively while they add destructively in $BR[B \to \eta' K^*]$ and so the OZI suppressed effects are felt more strongly in $B \to \eta' K^*$. Finally we present in the Table. 2 the results of

Table 2: Branching ratios(BR) for $B \to \eta(\eta')K(K^*)$ decays

Process	Experimental BR[24]	Theory BR
$B^- \to K^- \eta'$	$(80^{+10}_{-9} \pm 7) \times 10^{-6}$	93×10^{-6}
$B^- \to K^- \eta$	$< 6.9 \times 10^{-6}$	1.04×10^{-6}
$B^- \to K^{-*} \eta'$	$< 35 \times 10^{-6}$	3.6×10^{-6}
$B^- \to K^{-*} \eta$	$(26.4^{+9.6}_{-8.2} \pm 3.3) \times 10^{-6}$	10×10^{-6}

our calculation including all the terms in Eqn. 68 and Eqn. 69. We see from Table. 2 that our calculations are in reasonable agreement with experiment. In particular we note that the chirally enhanced contributions tend to further increase the branching ratio $BR[B \to \eta' K^*]$. From the table above we can calculate

$$\begin{aligned} R_K &= 0.01 \\ R_{K^*} &= 0.36 \end{aligned} \tag{70}$$

If experiments find that the ratio R_{K^*} is indeed much smaller than predicted here then this would indicate the presence of large non factorizable corrections that effectively cancel the OZI contributions as well as the chirally enhanced corrections.

6 Summary

We have considered the weak decays of a B meson to final states that are mixtures of S-wave radially excited components. We calculated nonleptonic decays of the type $B \to \rho'\pi/B \to \rho\pi$, $B \to \omega'\pi/B \to \omega\pi$ and $B \to \phi'\pi/B \to \phi\pi$ where ρ', ω' and ϕ' are higher ρ, ω and ϕ resonances. We found that the transitions to the excited states can be comparable or enhanced relative to transitions to the ground state. It would, therefore, be extremely interesting to test these predictions. We also studied the effect of radial mixing in the vector and the pseudoscalar systems generated from hyperfine interaction and the annihilation term. We found the effects of radial mixing to be small and generally negligible for all practical purposes in the vector system. However, in the $\eta - \eta'$ system the effects of radial mixing are appreciable and seriously affect decay branching ratios. In particular we found that the experimental violation of the sum rule for $B \to K\eta'$ in Eqn. 64 can be explained by radial mixing without need for the OZI suppressed transitions. We also pointed out that the place to look for an OZI suppressed contribution is in $B \to K^*\eta'$ decays where the the OZI suppressed transitions become important as OZI allowed term is small.

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